CONTENTS

Developing mathematics with Base Ten
iii
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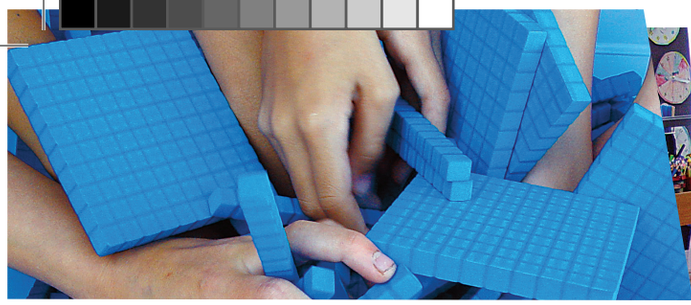
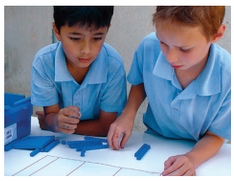




Foreword

For many decades, the use of manipulative materials to assist young children in their learning of mathematics has been recommended. The advocacy of great educators such as Maria Montessori, Zoltan Dienes and Catherine Stern encourages a wide acceptance of the use of manipulative materials, especially in primary school classrooms. Once, it was felt that simply giving students manipulatives to use in mathematics lessons would be enough to develop an understanding of mathematical concepts. This is not true. Manipulatives in and of themselves do not teach—skilled teachers do.

This series—*Hands-on mathematics*—is designed to help teachers who are trying to make the most of students' experiences with manipulatives. We believe it is better to use a few well-chosen products rather than an array of 'bits and pieces'. We recommend 'a lot of a little' rather than 'a little of a lot' when it comes to working with manipulatives. It is better to focus on a few well-chosen manipulative materials so that students will have an adequate supply of pieces. Nothing is more frustrating than not having enough to finish 'creating a design' or 'building that masterpiece'. As well, it is important that sufficient materials are available to allow models to be left on display in the classroom.

Frequently, when we work with students and teachers in classrooms and workshops, a range of common concerns is raised. Let us share a few with you.

Why use manipulatives?

When used as part of a well thought-out lesson, manipulatives can help students 'come to grips' with difficult concepts. The key to good use of manipulatives is for teachers to have a clear goal in mind when using them. This will help maintain the intention of the lesson and focus responses to any questions asked during the lesson. Teachers will have a clear idea of what to look for when observing students using manipulatives.

As Richard Skemp, the famous educational psychologist said, 'It is as though their thinking was out there on the table'.

We have observed how students experiment with ideas willingly. If, at first, satisfaction with an idea is not achieved, students will seek another solution. We do not see this happening as frequently when students are expected to work with abstract statements such as equations and written problems.

The skilled use of manipulatives—note, we said the *skilled* use of manipulatives—will enhance mathematics outcomes. Poor use may be detrimental to student attainment. This series of books is designed to ensure skilled use of manipulatives in the classroom.

Is there a difference between a mathematics manipulative and a mathematics teaching aid?

We believe there is a big difference between the two types of materials.

A child can interact, even take control of a good mathematical manipulative, whereas a teaching aid tends to control the learning experience. Too often, a teaching aid is used as a 'telling' support rather than a 'learning' support and experience has taught us that 'telling' is not a very successful method of teaching mathematical ideas.

How will I know whether the students are learning anything?

Observe the students as they work with the manipulatives. Don't worry if they solve a problem in a way different from what you expected. Ask questions. Encourage students to explain their thoughts or write about their experience.

In fact, actively engage with the students as their thought processes emerge. Simply using manipulatives is not enough. Students need to be given time to reflect on their activity and share their thoughts with a group or the whole class. The teacher plays a vital role in helping students connect new knowledge with old. Language plays a key role throughout this learning process.

What evidence can I show that students are learning or have learnt…?

Some teachers are concerned about the lack of written evidence to substantiate learning when manipulatives form a large part of the lesson. There are several ways a student might record his/her findings:

- *writing about the experience*
- *sketching or drawing any models produced*
- *photographing any models produced*
- *presenting 'learning tours' to students in other classrooms*
- *maintaining a learning journey logbook.*

Actually, when preparing this type of learning evidence, students have a wonderful opportunity to reinforce their own learning.

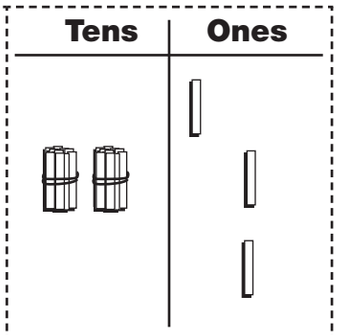
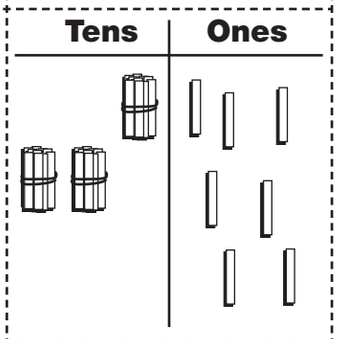
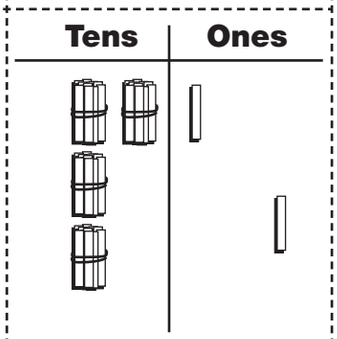
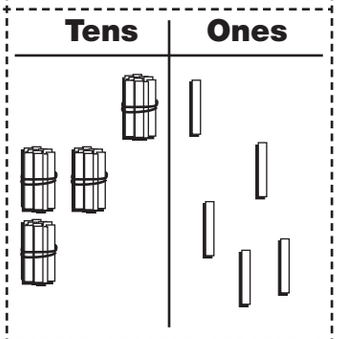
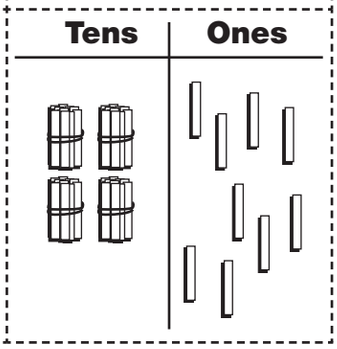
How do I manage the use of manipulatives?

Some teachers worry that students will only play with the manipulatives and not pay attention, or worse still begin to throw the material around. These are genuine fears which will decrease as experience, both by the students and teacher, increases. As with any 'new toy' there will be a 'novelty effect'. The first time you introduce a manipulative, allow time for the students to explore.

Set some simple rules and limits for the way the material is used and enforce these early on. Students will soon learn to respect the material.

Throughout this book, management ideas are presented. We encourage you to adopt them as your own.

Mix and match cards

23	2 tens 3 ones	Twenty-three	
37	3 tens 7 ones	Thirty-seven	
42	4 tens 2 ones	Forty-two	
45	4 tens 5 ones	Forty-five	
49	4 tens 9 ones	Forty-nine	

Decimal fractions

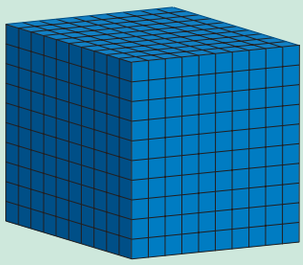
D T E S

Purpose

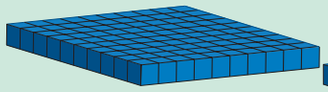
To develop the relationship between the Base Ten blocks further to model decimals.

Using Base Ten materials to model decimals

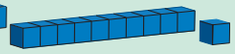
Note: mini represents 1.



cube
(represents 1000)



flat
(represents 100)



long
(represents 10)

mini
(represents 1)

A 10 x relationship exists between the mini and the long and then between the long and the flat and so on.

By changing the piece that represents 1 we can model decimals. Note: The relationship between the pieces remains the same.

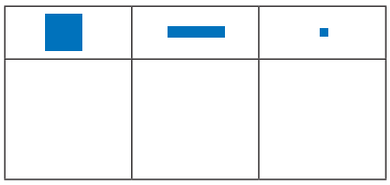
- If the flat represents 1, what would the value of the other pieces be? (long 0.1, mini 0.01)
- If the cube represents 1, what will each of the other pieces represent? (flat 0.1, long 0.01, mini 0.001)
- If the long represents 1, what will each of the other pieces represent? (cube 100, flat 10, mini 0.1)

Trading games using Base Ten materials may be played.

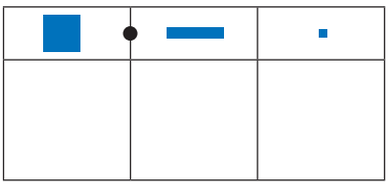
Note: Rather than a trading board with the following headings:

Hundreds	Tens	Ones

… we choose to use a trading board like this (see p. 44) …



… or this …



… with variations.

Background

Base Ten materials may be used to model decimal numbers. The transition can be fairly simple if the relationship between the pieces has been clearly established and the pieces have been referred to as 'long' and 'flat' rather than '10' and '100'.

If the flat is assigned a value of 1, then the long represents 0.1 or $^1/_{10}$ and the mini, 0.01 or $^1/_{100}$. The cube would represent 10.

D T E S

Subtraction

Purpose

To develop simple ideas of subtraction.

Situations leading to learning

There are many different situations that may lead to the need to subtract. While many people think primarily of 'taking away' when subtracting, there are many types of subtraction situations.

Consider the following situation: 'Aunty Sharon had 12 biscuits, then her children ate some. Now she has 7 biscuits. How many did the children eat?'

In this section, we will focus on the 'take away' notion of subtraction.

Encourage students to create many other situations which lead to the idea of subtraction.

From trading games to subtraction

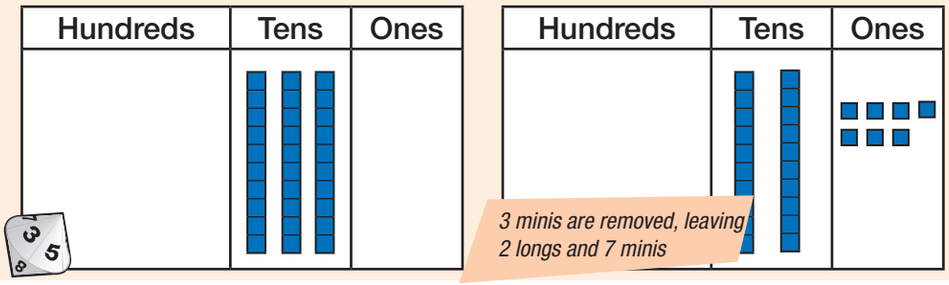
- Show me 4 less than a long. (Students will decompose the long to minis then remove 4 minis.)
- Remove 4 minis from this long.
- What is the difference between a long and 6 minis?
- What must I add to 6 minis in order to make a long?
- By how much are 4 minis placed end to end shorter than 1 long?
- What is left when I take 6 minis away from this long?

All of these situations may be solved using concepts of subtraction.

1 From trading to subtraction

Students will have practised this when playing trading games.

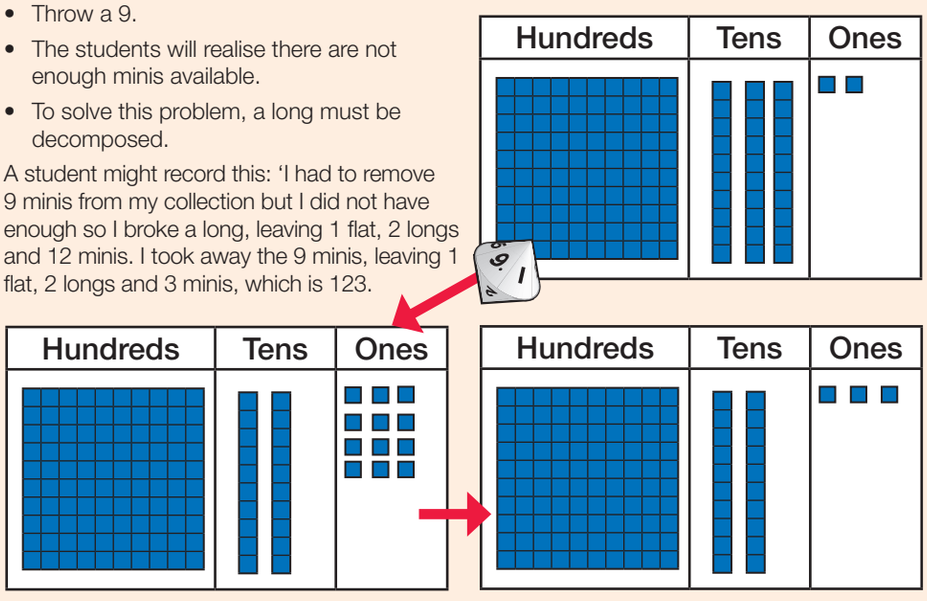
- Place a number of longs on the trading board.
- Throw a die. The number thrown must be removed from the board in minis.
- Remember to record the process.



2

- Begin with a collection similar to this.
- Use a 10- or 12-sided die.
- Throw a 9.
- The students will realise there are not enough minis available.
- To solve this problem, a long must be decomposed.

A student might record this: 'I had to remove 9 minis from my collection but I did not have enough so I broke a long, leaving 1 flat, 2 longs and 12 minis. I took away the 9 minis, leaving 1 flat, 2 longs and 3 minis, which is 123.



D T E S

Checking your work

Purpose

To develop techniques for estimating.

Background

When they are performing a calculation, we encourage students to follow a three-step process:

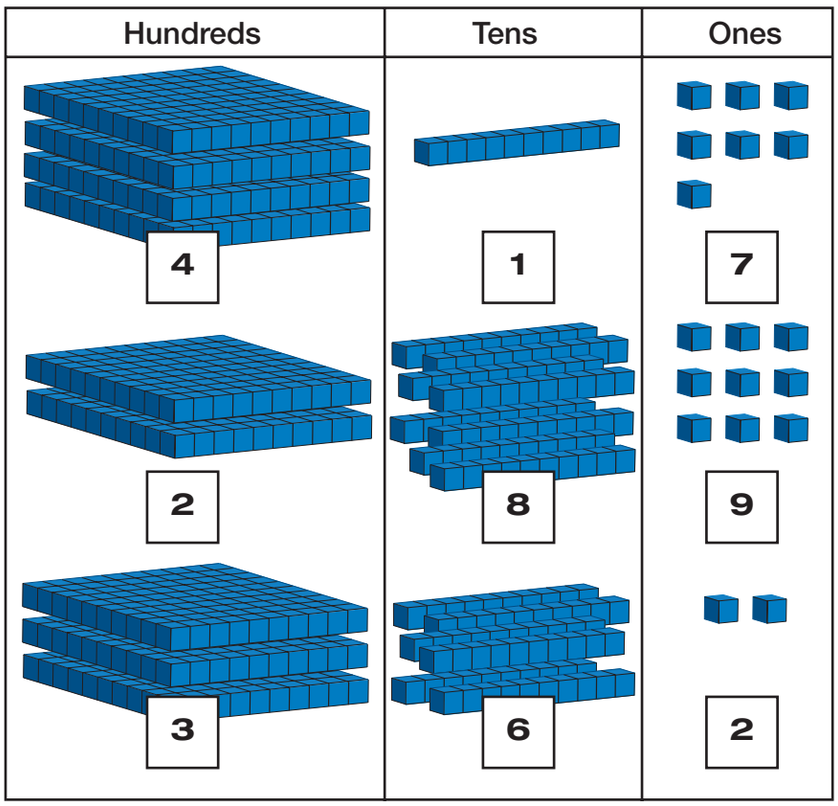
estimate—calculate—evaluate.

On this page we give a brief overview of some simple estimation techniques for addition. Front-end rounding methods—along with variations—may be applied to all four operations: +, −, x, ÷

Another simple checking technique uses patterns such as odd and even. For example, when adding two odd numbers we expect an even result.

When performing a calculation, students should be encouraged to make an estimate, which may be used to check the answer.

There are several techniques that may be used to form an estimate. For example, set out Base Ten blocks to show:



Discuss various ways you might estimate the result.

For example, you could cover all the tens and units and just focus on the hundreds

400 + 200 + 300 = 900

to form an estimate of 900.

Students will note that while this method is simple and easily executed, the estimate will always be below (under) the actual result.

Other students may note that the 89 part of 289 is close to one-hundred and so it would make sense to round 289 to 300. The estimate would then become 1000 which is much closer to the actual result.